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# A real time energy management strategy for plug-in hybrid electric vehicles based on optimal control theory

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## Abstract

Plug-in hybrid electric vehicles are commonly designed to work in Charge Depleting/Charge Sustaining (CD/CS) mode, depleting the battery by driving in only-electrical mode until the SoC reaches its minimum acceptable threshold, and then sustaining the state of charge till the end of the mission, operating as a traditional hybrid vehicle. Nonetheless, a simple application of an optimal control framework suggests a blended discharge strategy, in which the powertrain is operated as to gradually deplete the SoC and reach the lower threshold only at the end of the trip. Such an algorithm has the drawback that the optimal solution can only be reached offline, depending on the a-priori knowledge of the driving event, making it unsuitable to be implemented online, as it is. The paper presents a methodology to design a heuristic controller, to be used online, based on rules extracted from the analysis of the powertrain behavior under the optimal control solution. The application is a parallel plug-in vehicle, derived from a re-engineered engine-only driven powertrain, and the optimal problem is solved with the Pontryagin's Minimum Principle. Results are also compared to the same vehicle in its standard internal combustion engine version, as well as the commonly implemented Charge Depleting/Charge Sustaining strategy.

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**Keywords:** Energy Management, Plug-in, Rule-based, Pontryagin's Minimum Principle, Optimal Control;

## Nomenclature

$v$	Speed (km/h)	$\alpha$	Accelerator Pedal Position (%)
$t$	Time	$\beta$	Brake Pedal Position (%)
$T$	Torque (Nm)	$K$	Gain
$F$	Force (N)	$\omega$	Angular Speed (rad/s)
$P$	Power	$V$	Voltage (V)
$M$	Mass (kg)	$I$	Current (A)
		$Q$	Energy Capacity (As)
		$R$	Resistance

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$\eta$	Efficiency	$L$	Load
$SoC$	State of Charge (State Variable)	$veh$	Vehicle
$\dot{SoC}$	State of Charge Time Derivative	$des$	Desired
$\mathcal{J}$	Cost Function	$p$	Proportional
$u$	Control Vector	$i$	Integral
$\dot{m}$	Fuel mass flow rate	$eq$	Equivalent
$H$	Hamiltonian Function (kg/s)	$c$	Coulombic
$\lambda$	Co-state (kg)	$oc$	Open Circuit
$p$	Additive penalty function (kg)	$0$	Initial Value
$K$	Penalty weight value (kg)	$f$	Final Value
<b>Subscripts and Superscripts</b>		$fuel$	Fuel
$min$	Minimum Threshold	$*$	Optimal Solution
$max$	Maximum Threshold	$ICE$	Internal Combustion Engine
$wh$	Wheel	$EM$	Electric Motor
		$BP$	Battery Pack

## 1. Introduction

Plug-in hybrid electric vehicles (PHEVs) are seen as a promising industry solution for reducing fuel consumption and pollutant emissions. Compared to hybrid electric vehicles (HEVs), PHEVs have the advantage of a longer battery life, and obviously of rechargeability from an external source. Fully recharging the battery allows for an extension of the acceptable operating range of the state of charge (SoC), thus reducing PHEVs fuel consumption with respect to HEVs, with the additional consequence of producing less CO<sub>2</sub> emissions, [1], [2]. In total analogy with HEVs, the cumulative fuel economy of these vehicles depends in essence on the design of the powertrain components and on the choice of the architecture, [3], [4], but is particularly sensitive to the energy management control strategy implemented on-board, [5] and [6].

In PHEVs, the presence of a bigger battery which can be plugged to an external source to be restored to full charge introduces an additional degree of freedom beside conventional HEVs, since the larger allowable SoC range can be managed with different discharge strategies, [7]. So far, the Charge Depleting/Charge Sustaining (CD/CS) operation is still the most implemented strategy, being rather unrelated to the driving mission. In this strategy, the battery is initially discharged operating the vehicle as a pure electric vehicle, until the SoC reaches the final desired value, which is then sustained until the end of the trip. Nonetheless, a blended battery discharge, which gradually reaches the desired final SoC only at the end of the mission, results in the optimal energy management, and thus in the minimum fuel consumption and CO<sub>2</sub> emissions, [8]. The battery SoC is hence a crucial element in the choice of the best algorithm for the energy management of such vehicles. In general, if a powertrain has the possibility of operating in different modes, the SoC should not only affect the power split, but also the supervisory controller decision on the mode of operation, [9]. Several of the optimal control theory frameworks proposed for energy management of HEVs have been already applied to PHEVs, such as Dynamic Programming, [10], Equivalent Consumption Minimization Strategy, [11] or Pontryagin's Minimum Principle (PMP), [12], [13], [14] aiming at the minimization of different cost functions. Nevertheless, the main issue in the realization of a blended strategy implementable online lies in the need of an a-priori knowledge of a number of information, such as the total distance to be traveled and the characteristics of the driving cycle (speed and grade profiles), in order to correctly calibrate the optimization parameters and obtain the optimal battery SoC trajectory. On the other hand, a careful investigation on the powertrain behavior, when the optimal framework is applied, can help for the design of a satisfying heuristic control technique, which can lead to better results than the CD/CS strategy, being also independent on the driving mission. Similar studies have been already applied to HEVs, using for example the Dynamic Programming optimal algorithm as in [15].

In this paper a rule-based strategy is designed starting from the application of the PMP to a GM Chevrolet Malibu, which is originally driven only by an internal combustion engine (ICE). The aim of the study is to demonstrate the effectiveness of this rule-based strategy despite its naive implementation and low computational efforts. Also, the application to a re-engineered vehicle, aims at demonstrating the considerable fuel consumption reduction, while maintaining vehicle performance. In this application, the ICE has been coupled to an electric machine (EM) which

can operate both in motoring and regenerating modes, and the battery can be charged also during traction and not only during braking. Results obtained with the rule-based strategy are then compared to the optimal solution of the PMP, the results of a CD/CS strategy, as well as the fuel consumption of the ICE-only driven vehicle.

### 1.1. Powertrain architecture and vehicle modeling

The simulator used for the study is a quasi-static forward-looking model-based simulator developed in Simulink Matlab environment.

The driver model is based on a PID controller that compares the actual velocity of the vehicle,  $v_{veh}$  (which is a consequence of the torque delivered by the powertrain to the wheels) with the desired velocity,  $v_{des}$ . In this work, the controller has only the proportional ( $K_p$ ) and integral ( $K_i$ ) gains and the output is the accelerator ( $\alpha(t)$ ) or the brake ( $\beta(t)$ ) pedal position, as per Eq. (1):

$$\alpha(t) = K_p \cdot [v_{des}(t) - v_{veh}(t)] + K_i \cdot \int_0^t [v_{des}(t) - v_{veh}(t)] dt \quad (1)$$

where  $\beta(t) = -\alpha(t)$ , with the simulator choosing  $\beta(t)$  or  $\alpha(t)$  if the torque at the wheels is negative or positive, respectively.

The actual vehicle speed is calculated integrating the Newton Second law of motion, given by:

$$(M_{veh} + M_{eq}) \frac{dv_{veh}(t)}{dt} = F_{wh} - F_L \quad (2)$$

where the the road load force,  $F_L$ , takes into account the rolling resistance at tires, the aerodynamic resistance and the force due to the road slope. The equivalent mass,  $M_{eq}$ , is instead used to consider the rotational inertia of all the components of the driveline and is approximatively estimated in an increase of 10% of the vehicle mass,  $M_{veh}$ .

As already mentioned above, the vehicle used for the analysis is a Chevrolet Malibu, which has been re-engineered to become a PHEV with a parallel architecture. The original vehicle is equipped with a thermal engine LE5 of the GM Family II, whose specifications have been derived with a set of experimental tests realized at the Center for Automotive Research<sup>1</sup> and are listed in Table 1. Figure 1 a) shows the obtained brake specific fuel consumption (BSFC) map and the maximum torque envelope.

Table 1. Components Specifications.

Internal Combustion Engine		Electric Motor		Battery Pack	
Displacement	2.384 L	Rated Power	75 kW	Energy Capacity	13 kWh
No. Cylinders	4	Max/Min Peak Torque	270 Nm@3000-4200 rpm	Operating Voltage	340 V
Configuration	straight-4 pistons	Max/Min Rated Torque	130 Nm@0-5500 rpm	Max Charge Current	180 A
Compression Ratio	10.4:1			Min Discharge Current	-60 A
Fuel	Gasoline			SoC Range	0.95-0.25
Maximum Torque	176 Nm@5000 rpm				
Maximum Power	104 kW@6500 rpm				

The ICE is linked to the transmission shaft by a clutch and is coupled by a belt to the GVK210-150X permanent magnet electric machine (EM). The main specifications are listed in Table 1, while Fig. 1 b) portrays the efficiency map employed in the simulator.

The parallel powertrain is characterized by a pre-transmission architecture, which has been chosen for its simple and economically convenient implementation in re-engineered powertrains. The transmission is an automated manual transmission supervised by a logical controller. This control shifts the gears by acting on the signal of the accelerator pedal and the angular speed of the ICE and manages the clutch position during the shifting. In particular, by means of two maps, one for the up-shifting and another for the down-shifting, the gear is shifted in correspondence of a certain

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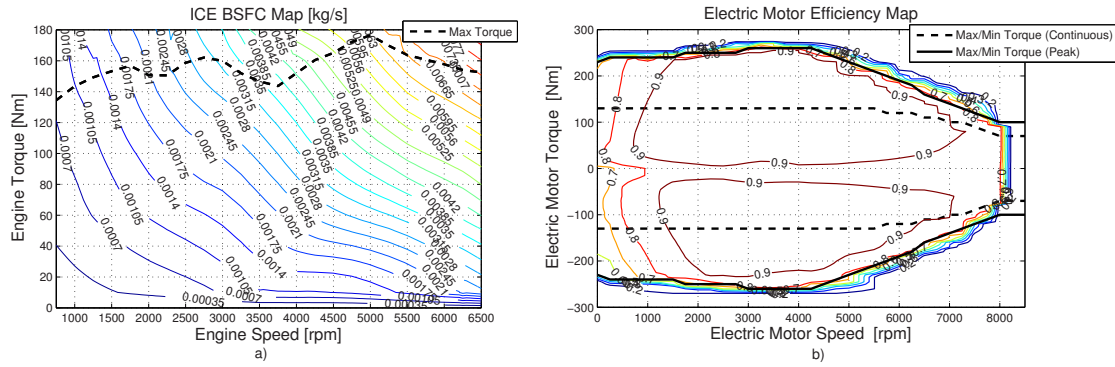


Fig. 1. a) Engine BSFC Map b) Motor Efficiency Map

engine angular speed that is an affine function of the accelerator position. In the present work, the shifting strategy is optimized for the only ICE operation. Nonetheless, being both the engine and the electric motor belted upstream the transmission shaft, a combined efficiency map could be computed to find the best gear ratios which allow for a combined operation of the propulsion systems in the highest efficiency area. This analysis is however out of scope for this paper and thus left to future studies.

The electric storage system is a 105S 2P battery pack of Li-Ion cells manufactured by A123. The pack main characteristics are given in Table 1.

The battery pack electrical dynamics are modeled using a zero-th order equivalent electric circuit model, by means of which the battery cell is represented by an electric circuit. Thus, the battery pack (BP) load voltage ( $V_{BP}$ ) follows the Kirchhoff law, as per Eq.(3):

$$V_{BP} = V_{oc}(SoC) - I_{BP} \cdot R_{eq}(SoC, T) \quad (3)$$

The open circuit voltage,  $V_{oc}$ , is, in principle, a function of the battery SoC, while the internal resistance,  $R_{eq}$ , is a function of both SoC and temperature. These dependences are taken into account by specific maps and a simple thermal model for the battery, which converts the battery power losses into heat power. In this study, the battery temperature is only used to update the zero-th order battery model and not taken into account in the control problem formulation, therefore the thermal model is not shown in details. This choice is justified by the consideration that the variation of those parameters with the temperature is generally small for Li-Ion batteries and usually neglected. For a smooth reading, these explicit notations will be dropped in the following, such as the explicit dependence of variables on time.

## 1.2. Operating Modes

In the following, the modes of operation allowed for the specific powertrain architecture, during traction, are singled out:

1. **Electric mode (EV):** the drive torque is supplied exclusively by the EM, the ICE is turned off and disconnected to the clutch. The operating mode has no degrees of freedom, since the EM must satisfy all the driver's torque request, within its physical limits.
2. **Regenerating mode (RV):** the ICE torque is greater than the driver's torque demand and the *surplus* is recovered by the EM to charge the battery. The clutch is engaged. The operating mode has one degree of freedom consisting in the extra amount of ICE torque.
3. **Power Split mode (PSV):** the EM and the ICE are connected together to the transmission to supply positive torque to the wheels. Thus, the clutch is engaged and the mode of operation is characterized by one degree of freedom consisting in the torque split between the propulsion systems.

It is worth pointing out that the regenerative braking is also implemented, where the electric machine recovers the kinetic energy otherwise wasted.

## 2. Pontryagin's Minimum Principle as a solution of the optimal control problem in PHEVs

The minimization of fuel consumption along a route is usually taken as the main goal for the design of the energy management strategy of a hybrid vehicle. In particular, this optimization problem lies in finding the optimal trajectory  $u^*(t)$ , which minimizes an integral cost function,  $J$ , defined over the entire optimization horizon, which corresponds to the length of the driving cycle:

$$\min_u J = \int_{t_0}^{t_f} \dot{m}_f(u(t), t) dt \quad (4)$$

The control vector,  $u(t)$ , is composed by a number of control variables depending on the degrees of freedom of the energy paths. Thus, it represents a measure of the power split between the power systems on-board. As such a problem is characterized by only one degree of freedom, the control function can be seen as the power delivered by the battery pack, directly related to the power split between the EM and the ICE, such as:  $u(t) = P_{BP}(t)$ .

If the fuel consumption minimization is the only optimization target, then the system can be treated as quasi-static and the only state variable is the state of charge and the state variable dynamics is given by:

$$S \dot{O}C = -\eta_c \frac{I_{BP}}{Q_{BP}} \quad (5)$$

where the Coulombic efficiency,  $\eta_c$ , counts for charge losses. It is useful to express the state variable dynamics as a function of the state variable itself and the control variable, such as:  $S \dot{O}C = f(SOC, P_{BP})$ .

Thus, by using Eq. (3), the battery power can be expressed as follows:

$$P_{BP} = V_{BP} \cdot I_{BP} = V_{oc} \cdot I_{BP} - I_{BP}^2 \cdot R_{eq} \quad (6)$$

and then, by solving Eq. (6) for  $I_{BP}$ , the current at battery terminals (positive in discharge and negative in charge) can be expressed as a function of the battery power and of the SoC (by means of  $V_{oc}$  and  $R_{eq}$ ), and can be used in Eq. (5).

The minimization problem is also subject to some instantaneous constraints, that must be met at very instant of time, given by the physical limits of the battery and the other components, as well as the drivability constraint of fulfilling the torque demanded by the driver, over any given trip. Moreover, since for a PHEV the optimal control scheme gives a blended mode of operation, for sake of optimality, the boundary conditions on the SoC dynamics must satisfy Eq. 7:

$$\begin{cases} SOC(t = t_0) = SOC_0 \\ SOC(t = t_f) = SOC_f \end{cases} \quad (7)$$

Where  $SOC_0$  and  $SOC_f$  are generally imposed equal to the maximum and minimum allowable states of charge, respectively.

The optimization problem, defined above, can be tackled by solving the Pontryagin's Minimum Principle, which defines the Hamiltonian function,  $H$ , as:

$$H(SOC, P_{BP}, \lambda(t)) = \dot{m}_{fuel}(P_{BP}) + \lambda(t) S \dot{O}C(SOC, P_{BP}) \quad (8)$$

The PMP provides a set of necessary conditions to be respected by the optimal control sequence,  $P_{BP}^*$ , which minimizes the fuel consumption,  $\dot{m}_{fuel}(P_{BP})$ , achieving the optimal SoC trajectory,  $SOC^*$ . In particular the optimal control variable, for each time step, must satisfy the following condition:

$$H(SOC, P_{BP}, \lambda(t)) \geq H(SOC^*, P_{BP}^*, \lambda(t)^*) \quad (9)$$

where  $\lambda(t)$  is the co-state which varies with time and whose optimal trajectory,  $\lambda^*(t)$ , must satisfy the following dynamic equation:

$$\dot{\lambda}^* = -\frac{\partial H(SOC^*, P_{BP}^*, \lambda^*)}{\partial SOC} = -\lambda^* \frac{\partial S \dot{O}C(SOC^*, P_{BP}^*)}{\partial SOC} \quad (10)$$

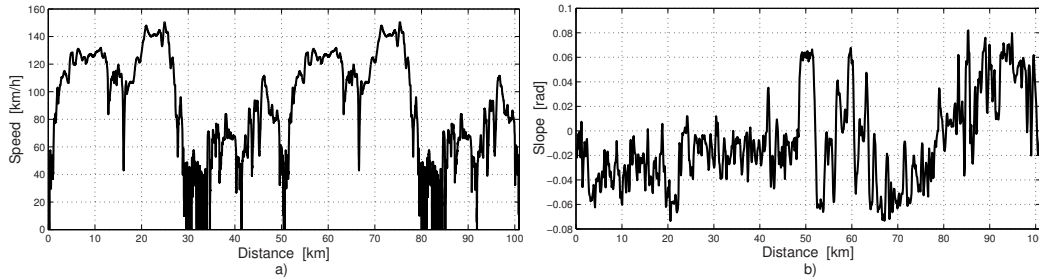


Fig. 2. Mixi speed a) and grade b) profiles

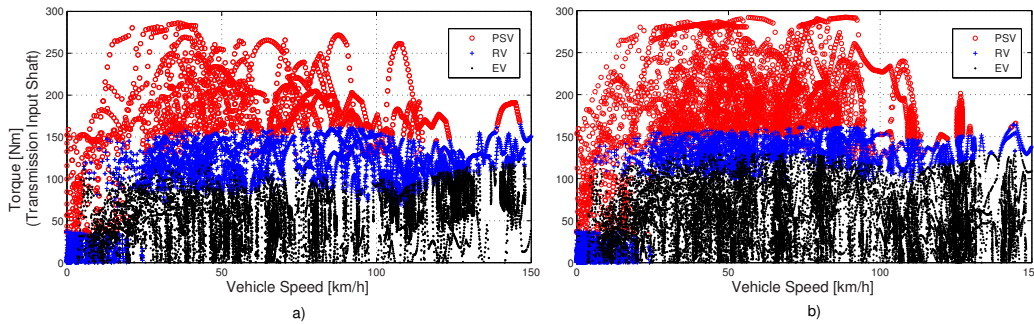


Fig. 3. Optimal distribution of the operating modes for the Mixi without grade a) and with grade b)

Since a unique trajectory satisfies the boundary conditions, thus, this set of conditions is also sufficient for the global optimal control, [16], and the PMP can be used as a global optimization tool.

In the rest of the paper, the \*, denoting the optimal solution, is dropped as we consider only the optimal control policy.

### 3. Analysis of the optimal powertrain behavior

A set of simulations has been performed to investigate the behavior of the powertrain under the application of the optimal control tool. The simulation set is composed by:

1. a pattern consisting of a sequence of three standard cycles, namely Urban, Extra-Urban and Highway Artemis. This driving cycle, hereafter called Mixi, has been simulated with and without considering road grade variations;
2. a real cycle, called Arco Merano, obtained with a GPS data acquisition, simulated with and without the road grade, as for the previous case;
3. several other standard driving cycles, such as FUDS, FHDS, US06 and compositions of them.

All the driving cycles have been simulated for total distances of around two times the all electric range (AER) of the vehicle, with respect to the given driving cycle. The AER represents the distance the vehicle can cover by using exclusively the energy available from the battery and keeping the ICE off. In particular, the chosen distance interval is of 100-250 km (see Table 2), since in [17] it has been observed that the fuel consumption has an increasing trend with the trip distance, tending towards an asymptotic value for long distances (greater than 200 km), confirming that over large distances a PHEV behaves asymptotically like a HEV and its choice is no more justified.

For sake of fluency, results of this analysis are though presented only for the Mixi driving cycle, with and without considering altitude variations, see Fig. 2. In order to analyze the behavior of the powertrain, the modes of operation chosen by the PMP over this cycle have been plotted as a function of the transmission input shaft torque and the vehicle speed, as shown in Fig. 3.

One can observe that three horizontal bands as a function of the pre-transmission torque can be identified. In fact, Fig. 3 shows that the optimal solution of the PMP, for torques lower than about 100 Nm makes the powertrain operate



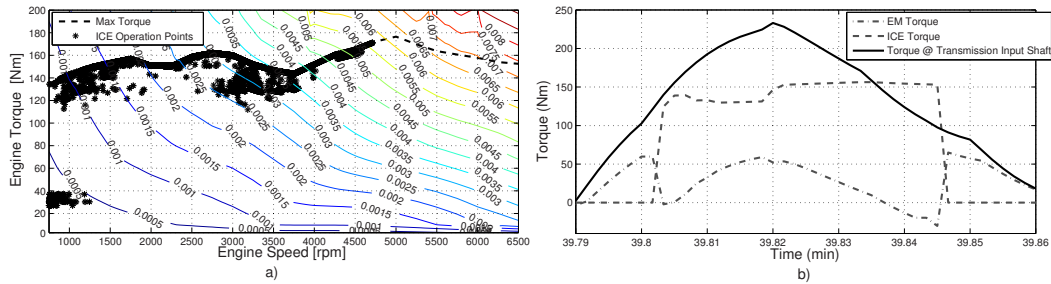


Fig. 4. Optimal distribution of the ICE operating points for the Mixi w/o grade a) and a particular of the optimal torque split b)

in EV, while for torques between approximately 100 Nm and 150 Nm the powertrain works in RV. Instead, for torques greater than about 150 Nm, the selected mode of operation is PSV.

This can be explained looking at the engine BSFC and the EM efficiency maps in Fig. 1. In fact, the EM efficiency is, in the feasible operating area, almost constant and equal to the maximum value. This implies that for the electric machine, both in traction (PSV) and RV, almost all the points in the map are eligible for being selected by the optimizer. On the other hand, the engine map<sup>2</sup> is such that, once the engine speed is imposed by the gear shifting strategy, the fuel consumption is not particularly affected by changes in torque. It is convenient recalling that the PMP assures global optimality under the constraint of achieving a blended discharge strategy, and thus the optimal value of the co-state is the one that makes the battery deplete gradually. In some way, this means that the PMP, when the optimal co-state is found, can foresee the future driving conditions and can optimally decide when the battery must be depleted and when it must be recharged. Given that, since the maximum feasible EM torque in traction is lower than 130 Nm, the optimization algorithm makes the powertrain use only the electric machine for low torques and whichever speed, working in EV mode. Instead, for greater torque requests, the ICE supplies a torque near to the maximum value available at the speed imposed by the gear shifting strategy, as shown in Fig. 4 a) and, particularly, in Fig. 4 b). This behavior results from the necessity of recharging the battery, to restore the energy used in the EV mode so that the desired final SoC is reached only at the end of the driving cycle. Thus, the ICE provides torque both for traction and for battery charging. On the other hand, it is easy to understand that for very high torques (beyond 150 Nm) the powertrain is forced to work in PSV to satisfy the driver's torque request. In this operating mode, the ICE torque is regulated by the motor torque as a function of the efficiency maps, as clearly depicted by Fig. 4 b), explaining the presence of several engine operating points below the maximum torque envelope in Fig. 4 a).

Figure 3 shows that in presence of a varying altitude profile the optimal powertrain behavior given by the PMP is practically equivalent to the results obtained in the case without grade. The only difference can be observed in a certain overlap between the EV and RV operating points distribution. This can be explained considering that the road grade profile, being characterized by several downhill stretches strongly helps the powertrain recharge the battery and therefore the EV mode is more often allowed. The same figure shows a low torque/low speed region where it is not possible to find any clear correlation between the mode selection of the PMP and the driving conditions. Thus, being not possible identifying a well-defined area for each operating mode, this behavior is not reproduced in the further rule-based strategy. In particular, the dependence on the speed is not taken into account, basing the design of the strategy only on torque criteria, with the great practical advantage of having only the drive torque as output signal from the controller.

The simulations performed with the other driving cycles have confirmed this behavior of the powertrain as driving conditions change. This could also be explained considering that the powertrain characteristics (*i.e.* the EM and ICE maps) and the battery size are such that the optimal value of the co-state results in being practically independent on the driving cycle features, as summarized in Table 2 for some of the simulated driving cycles. The standard deviation of all these co-state values is of only 0.0266 kg and this helps understand why the obtained powertrain behavior is very similar for any driving condition and justifies the design of a rule-based strategy, guaranteeing reliability and good results.

<sup>2</sup> It is worth recalling that this map is derived from bench tests carried on at OSU CAR laboratories

Table 2. Optimal initial co-state values for the simulated driving cycles.

Driving Cycle	Length [km]	$\lambda_{opt}$ [kg]	Driving Cycle	Length [km]	$\lambda_{opt}$ [kg]
Mixi w/ grade	100.92	3.187	FUDS	144	3.247
Mixi w/o grade	100.92	3.215	FHDS	165	3.276
Arco Merano w/ grade	224.8	3.235	US06	130	3.224
Arco Merano w/o grade	224.8	3.234	FUDS+FHDS	142.5	3.267

#### 4. Rule-based strategy design

Some rules can be therefore extracted from this first analysis. In particular, the realized rule-based controller first selects the operating mode on the basis of the wheel torque request and then a torque split between the EM and the ICE is imposed when needed, i.e. when there is at least one degree of freedom.

Since an uncertainty has been observed in the threshold between the EV and the RV modes, a sensitivity analysis has been performed to find the best threshold value to switch from a mode of operation to the other. A torque of 105 Nm is the one that allows achieving better results in terms of fuel consumption in most of the driving cycles. Instead, the threshold value between the RV and the PSV modes has been set to 140 Nm, which is a value always lower than the maximum ICE torque envelope, in the operating speed range of the ICE, which guarantees having some extra torque to charge the battery, when the RV mode is selected. For greater torques the EM is used in traction, together with the ICE to satisfy the extra demand. The degree of freedom on the torque split, in the RV and the PSV modes, has been suppressed by making the ICE work on its maximum torque envelope, as suggested by the PMP.

Moreover, since there is no guarantee that the resulting SoC profile does not drop under the minimum allowable SoC value (30%) or overcomes the maximum allowable SoC value (95%), this occurrences must be prevented. Therefore, when the battery SoC level is too low (*i.e.* under 30%), the vehicle is forced to run in RV mode in order to restore the battery SoC to higher values, and, in particular, the vehicle starts working in charge sustaining mode until the end of the driving cycle. Similarly, in order to avoid the risk of overcharging the battery, if the SOC is greater than 94%, only the EV mode is allowed.

#### 5. Validation of the RB strategy

The effectiveness of the developed rule-based strategy has been first verified reproducing some of the driving cycles used in the first phase of the optimal solution analysis. Then, the strategy has been validated by simulating the vehicle behavior over driving cycles different from the ones employed in the calibration phase. In particular, a high demanding mountain driving cycle, called VAIL2NREL, and a realistic driving cycle derived from a GPS acquisition and reproducing a pattern of the city of Aachen, therefore called Aachen, have been simulated. Those cycles have been chosen for their significant differences from the driving cycles used for the calibration process. The proposed results are also compared to the same powertrain working under a CD/CS strategy, which is commonly implemented on PHEVs, and to the fuel consumption obtained with the original ICE-only driven powertrain, in order to verify the effectiveness of the re-powering.

The CD/CS strategy has been modeled using the same PMP algorithm where the co-state has been first set equal to zero, which virtually assigns no costs to the use of the battery, to force using only the electric machine and realize a charge depleting phase. When the SoC reaches the value of 30%, a weighting factor is summed to the co-state in the Hamiltonian function of Eq. (8), virtually increasing the cost of the battery usage, in order to keep the SoC within its boundary limits, as suggested in [5].

Table 3 provides the results, in terms of fuel consumption, obtained with the rule-based strategy compared to the optimal solution of the PMP, the CD/CS strategy and the fuel consumption of the ICE-only driven vehicle. It is worth noting that the fuel consumption obtained with the conventional vehicle, before the re-powering, is on average equal to 11 km/l. This high value for the consumption mainly explains the need of a re-powering and justifies the high percentage value of fuel savings shown in the table. As expected, the rule-based strategy cannot perform better than the optimal PMP strategy, but always gives better results than the CD/CS strategy. In fact, simulating driving cycles which are completely different from the ones employed in the calibration phase can give results much worse or much



Table 3. Rule-based (RB) range comparison [km/l] in percentage w.r.t. PMP, CD/CS and ICE-only driven vehicle

Driving Cycle	RB vs PMP	RB vs CD/CS	RB vs ICE-only
Mixi w/ grade	-8.97%	+6.01%	+52.40%
Mixi w/o grade	-4.72%	+8.16%	+54.34%
Arco Merano w/ grade	-6.04%	+9.01%	+40.03%
Arco Merano w/o grade	-7.19%	+5.94%	+52.99%
VAIL2NREL	-0.29%	+9.59%	+63.24%
Aachen	-17.5%	+3.00%	+52.80%

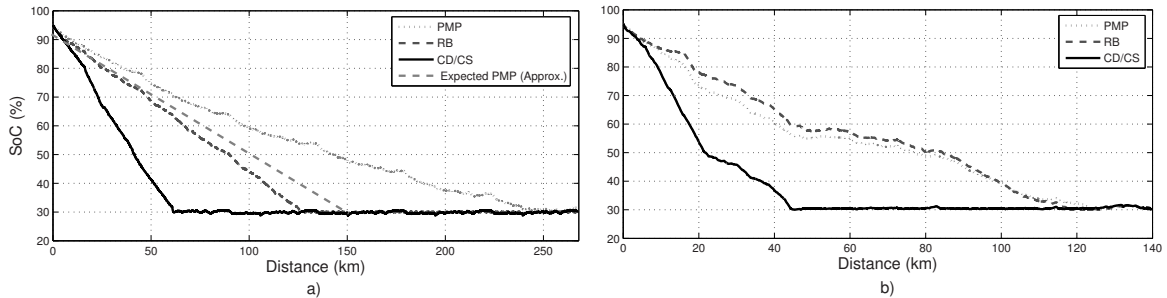


Fig. 5. SoC profiles for the Aachen w/o grade a) and for the VAIL2NREL w/ grade b)

better than the ones obtained with the other driving cycles, if compared to the optimal solution. The fuel consumption in the case of the VAIL2NREL cycle, which is a high demanding mountain driving cycle (average speed of around 85 km/h and slope varying between  $\pm 8\%$ ), results in being very close to the optimal one. On the contrary, the Aachen cycle, which is a urban driving cycle, characterized by a low average speed (around 42 km/h) and a flat pattern, results in being a low demanding driving cycle and gives results quite far from the optimum, even if still better than the CD/CS algorithm. The great difference in behavior of the two driving cycles can be understood by looking at the obtained SoC profiles in Figure 5. Figure 5 b) shows how the SoC profile of the rule-based strategy is, in the case of the VAIL2NREL, almost superimposed to the optimal SoC profile, explaining the value of the fuel consumption very similar to the solution of the PMP in Table 3. On the other hand, Fig. 5 a) shows that the SoC profile obtained with the rule-based strategy for the Aachen cycle has a pronounced charge sustaining phase. This depends on that this cycle is the longest one over which the simulator has been tested and thus, the optimal value of the co-state is most likely higher than the ones obtained for the other driving cycles. Nevertheless, as explained above, for such distances, a PHEV could be no more suitable and the correct choice should be a traditional HEV. Figure 5 a) also shows an approximation of the expected optimal SoC profile for the same driving cycle under a total driven distance of around 150 km, which could still be a reasonable value for choosing a PHEV. There is no reason to believe that the rule-based (or the CD/CS) SoC profiles should change, changing the driven distance, as those strategies are both independent on the driving conditions. Thus, one can note that for such a distance the rule-based SoC profile tends to be quite near to the optimal one, which would be most likely reflected on a fuel consumption close to the optimal value.

## 6. Conclusions

The paper has presented a methodology to design an online implementable strategy based on the optimal control theory. In particular, the methodology has involved an in-depth analysis of the behavior of the powertrain under the application of the optimal tool, carried on over a variety of driving cycles, strongly different in terms of average speed, maximum speed, grade profile and total distance to be driven. This careful calibration phase has been a crucial step to allow extracting rules suitable for the design of an effective controller. The realized rule-based strategy has been then demonstrated to achieve a fuel consumption somehow in between of the optimal solution and the CD/CS solution, giving results always better than the CD/CS paradigm. Nevertheless, simulations of driving cycles very different from the ones employed in the design phase have shown the possibility that this algorithm gives results either very close to the optimal ones or very far from them. Therefore, the effectiveness of the strategy, in the meaning of ability in

giving sub-optimal results, cannot be guaranteed under any driving condition, and the main disadvantage resides in being not flexible with respect to the distance. In general, it can be expected that, for very long driving cycles, the controller will still be able to achieve a better fuel economy than the one obtainable with the CD/CS strategy but quite far from the optimal solution while, for very short driving cycles, and namely shorter than the AER of the vehicle, this strategy will not be able to work in EV mode as one would desire. It is worth saying that the strategy has been though realized for a re-powered vehicle and thus the primary objectives were reliability, simple implementation and low computational efforts. The possibility of designing the energy management strategy iterating its calibration with the components sizing optimization could most likely achieve better results.

Future developments can surely look at the possibility of optimizing the gear shifting strategy for the whole pre-transmission system composed by the engine and the electric machine, but another interesting investigation could focus on correlating the rule-based strategy with information about the driving path in order to achieve good results also over driving cycles very different one to another. The driving path information can be obtained from GPS systems or ITS (Intelligent Transportation Systems), providing information such as the total distance of the trip, the upcoming grade profile and the foreseen velocity profile. All these information can be correlated to SoC discharge profile and could help make the controller adaptive to the specific path, giving the possibility of reaching a better fuel economy.

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